

Evaluate the integral $(\cos 3A)^8 dA$ from 0 to $\pi/6$.

Solutions

$$\int_0^{\pi/6} \cos 3A \, dA$$

Let: $u = 3A$; $du = 3dA$ thus, $dA = du/3$

Change limits:

when $A = 0$, $u = 0$; at $A = \pi/6$, $u = \pi/2$

Substitute:

$$\int_0^{\pi/2} \cos^8 u \frac{du}{3} = \frac{1}{3} \int_0^{\pi/2} \cos^8 u \, du$$

Using Wallis formula: $n = 8$ and $\alpha = \pi/2$, since n is an even number

$$\frac{1}{3} \int_0^{\pi/2} \cos^8 u \, du = \frac{1}{3} \left(\frac{7(5)(3)(1)}{8(6)(4)(2)} \right) \left(\frac{\pi}{2} \right)$$

$$= \frac{105 \pi}{2304}$$

$$= \frac{35 \pi}{768}$$