

1.)

Solve for x in the question:  $\text{Arc tan } 2x + \text{Arc tan } x = (\pi / 2) - 45^\circ$

*Solutions*

$$\begin{aligned}\text{Let: } A &= \text{Arc tan } 2x \\ \tan A &= 2x\end{aligned}$$

$$\begin{aligned}B &= \text{Arc tan } x \\ \tan B &= x\end{aligned}$$

$$\text{Arc tan } 2x + \text{Arc tan } x = (\pi / 2) - 45^\circ$$

$$A + B = 90^\circ - 45^\circ$$

$$A + B = 45^\circ$$

Take tangent on both sides:

$$\tan (A + B) = \tan 45^\circ$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B = 1 - \tan A \tan B$$

$$2x + x = 1 - 2x(x)$$

$$3x = 1 - 2x^2$$

$$2x^2 + 3x - 1 = 0$$

Using the formula:

$$x = \frac{3 \pm \sqrt{(3)^2 - 4(2)(-1)}}{2(2)} = \frac{-3 \pm 4.123}{4}$$

$$x = \frac{-3 + 4.123}{4} = 0.281$$

$$x = \frac{-3 - 4.123}{4} = -1.781$$

2.)

If  $84^\circ - 0.4x = \text{Arc tan}(\cot 0.25x)$ , find  $x$ .

*Solutions*

$$84^\circ - 0.4x = \text{Arc tan}(\cot 0.25x)$$

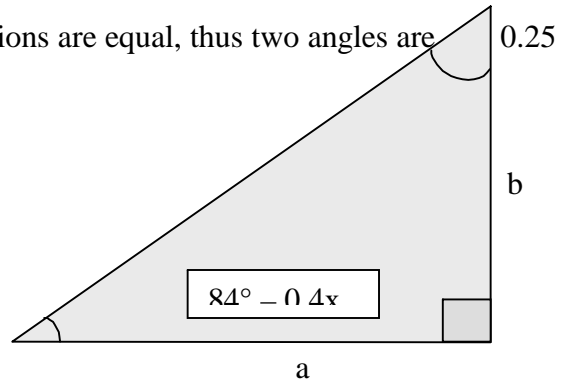
$$\tan(84^\circ - 0.4x) = \cot 0.25x$$

Since the tangent and the cotangent functions are equal, thus two angles are complimentary.

$$(84^\circ - 0.4x) + 0.25x = 90^\circ$$

$$0.15x = 6^\circ$$

$$x = 40^\circ$$



3.)

$$\text{Evaluate } \cos \left[ \arctan \frac{15}{8} - \arctan \frac{7}{24} \right]$$

*Solutions*

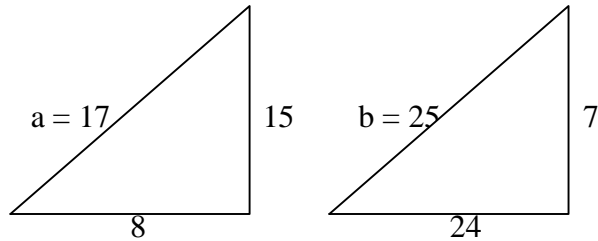
Let:

$$A = \arctan 15 / 8$$

$$\tan A = 15 / 8$$

$$B = \arctan 7 / 24$$

$$\tan B = 7 / 24$$



Using Pythagorean theorem:

$$a = \sqrt{8^2 + 15^2} = 17$$

$$b = \sqrt{24^2 + 7^2} = 25$$

$$\begin{aligned} \cos \arctan \left[ \frac{15}{8} - \arctan \frac{7}{24} \right] &= \cos (A - B) \\ &= \cos A \cos B + \sin A \sin B \end{aligned}$$

Refer to the figure, substitute values:

$$\begin{aligned} \cos A \cos B + \sin A \sin B &= \left( \frac{8}{17} \right) \left( \frac{24}{25} \right) + \left( \frac{15}{17} \right) \left( \frac{7}{25} \right) \\ &= \frac{297}{425} \end{aligned}$$

4.)

A merchant has three items on sale namely: a radio for \$50,00, a clock for \$30,00, and a flashlight \$1,00. At the end of the day, she has sold a total of 100 of the three sale items and has taken exactly \$1,000.00 on the total sales. How radios did she sell?

### *Solutions*

Let:  $x$  = number of radios sold  
 $y$  = number of clock sold  
 $z$  = number of flashlight sold

$$x + y + z = 100$$

$$z = 100 - x - y \rightarrow \text{Equation 1}$$

$$50x + 30y + z = 1000 \rightarrow \text{Equation 2}$$

Substitute Eq. 1 in Eq. 2:

$$50x + 30y + 100 - x - y = 1000$$

$$49x + 29y = 900 \rightarrow \text{Equation 3}$$

### *Notes*

The equation is a Diophantine equation. The value of  $x$  and  $y$  must be whole numbers. Try to substitute values of  $x$  from choices in Eq. 3 to solve for  $y$  (whole number)

when  $x = 16$ :

$$49x + 29y = 900$$

$$y = \frac{900 - 49x}{29}$$

$$= \frac{900 - 49(16)}{29}$$

$y = 4 \rightarrow$  whole number

Thus, the number of radios to be sold must be 16.