

1.)

The arithmetic mean of 6 numbers is 17. If two numbers are added to the progression, the new set of numbers will have an arithmetic mean of 19. What are the two numbers if their difference is 4?

Solutions

Let: x = the first number
 $y + 4$ = the second number
 y = sum of the original 6 number

$$\text{Arithmetic mean} = \frac{y}{6}$$

$$17 = \frac{y}{6}$$

$$y = 102$$

$$\frac{y + x + (x + 4)}{6 + 2} = 19$$

$$\frac{102 + 2x + 4}{8} = 19$$

$$106 + 2x = 19(8)$$

$$2x = 46$$

$$x = 23 \rightarrow \text{first number}$$

$$x + 4 = 27 \rightarrow \text{second number}$$

2.)

The distance between the points AB defined by A (cos A, -sin A) and B (sin A, cos A) is equal to

Solutions

Using distance formula:

$$d = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

$$d = \sqrt{(\sin A - \cos A)^2 + (\cos A + \sin A)^2}$$

$$d^2 = (\sin A - \cos A)^2 + (\cos A + \sin A)^2$$

$$d^2 = \sin^2 A - 2 \sin A \cos A + \cos^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A$$

$$d^2 = 2 \sin^2 A + 2 \cos^2 A$$

$$d^2 = 2 (\sin^2 A + \cos^2 A)$$

$$d^2 = 2 (1)$$

$$d = \sqrt{2}$$

3.)

What nominal rate, compounded semi-annually, yields the same amount as 16% compounded quarterly?

Solutions

$$\text{ER}_{\text{semi-annual}} = \text{ER}_{\text{quarterly}}$$

$$\left(1 + \frac{\text{NR}}{2}\right)^2 - 1 = \left(1 + \frac{\text{NR}}{4}\right)^4 - 1$$

$$\left(1 + \frac{\text{NR}}{2}\right)^2 = \left(1 + \frac{0.16}{4}\right)^4$$

$$\left(1 + \frac{\text{NR}}{2}\right)^2 = 1.1698556$$

$$1 + \frac{\text{NR}}{2} = 0.1632 \text{ or } 16.32\%$$

4.)

If $A + B + C = 180^\circ$ and $\tan A + \tan B \tan C = 5.67$, find the value of $\tan A \tan B \tan C$.

Solutions

$$A + B + C = 180^\circ$$

$$A + B = 180^\circ - C$$

Take tangent on both sides:

$$\tan(A + B) = \tan(180^\circ - C)$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Substitute:

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\tan 180^\circ - \tan C}{1 + \tan 180^\circ \tan C}$$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\text{As given, } \tan A + \tan B = \tan C = 5.67$$

$$\text{Thus, } \tan A \tan B \tan C = 5.67$$

5.)

The volume of the sphere is increasing at the rate of $6 \text{ cm}^3/\text{hr}$. At what rate is its surface area increasing (in cm^2/hr) when the radius is 50 cm ?

Solutions

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dv}{Dt} = \left(\frac{4\pi}{3} \right) (3r^2) \left(\frac{dr}{dt} \right)$$

Substitute values:

$$6 = 4\pi (50)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = 0.000191$$

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = (4\pi) (2r) \frac{dr}{dt}$$

$$\frac{dA}{dt} = 0.24$$

6.)

How much must you invest today in order to withdraw P 2,000 annually for 10 years if the interest rate is 9%?

Solutions

$$P = \frac{A [(1 + i)^n - 1]}{(1 + i) ni}$$

$$P = \frac{2000 [(1 + 0.09)^{10} - 1]}{(1 + 0.09)^{10} (0.09)}$$

$$P = 12,835.32$$